Recovery Mechanism in Machine repair system with Cobot and Reboot Impact

Sapna

Manipal University Jaipur, Jaipur, Rajasthan-303007, India

Anamika Jain

RD & DJ College Munger, Munger University, Bihar-811201, India

Abstract: This is an investigation of M/M/2 heterogeneous server, queueing model in which first server is totally reliable and always available but the second server provides two services. The second server originates with two services, one is an essential service (ES) and the second is optional service (OS). The arrival of failed units follows a Poisson distribution and the service follows an exponential distribution. The second server is the cobot server which is a combination of humans and robots and provides two services. In the development of robotic services, these robots face difficulty in real-time tracking, so reboot and recovery are also included. The proposed model is developed with two servers and working vacation. A prospective implementation of proposed model is also provided. The state-transition diagram and the corresponding differential-difference equations for the given system are introduced. R-K IVth is used to obtain system performance measurements. Numerical results and graphs are displayed by MATLAB and using ANFIS adaptive neuro-fuzzy inference system for verify the computational tractability of the expected queue length.

INTRODUCTION

Queueing theory helps improve machine repair models by analyzing and optimizing the flow of repair tasks, ensuring efficient resource allocation and minimizing downtime. By modeling the repair process as a queue, it becomes possible to predict wait times, identify bottlenecks, and optimize the number of repair technicians needed. For instance, queueing theory can help determine the optimal number of spare parts to keep in stock or the best schedule for maintenance personnel to minimize idle time and ensure timely repairs. This leads to a more balanced workload and prevents delays in repairing machines, enhancing overall productivity. Furthermore, queueing theory enables better decision-making by providing insights into the performance and reliability of the repair system under different conditions. It can simulate different scenarios, such as an increase in machine breakdowns or changes in repair times, helping organizations plan for breakdowns. By understanding these dynamics, businesses can implement different type of servers, such as essential server and optional server. Overall, queueing theory aids in creating a robust and responsive machine repair system that can adapt to changing demands and maintain operational efficiency. At first there is some literature review about heterogeneous server.

Many authors investigate in heterogeneous servers An et al. (2021) investigate about the flexible scheduling in maintains and repairman under maintenance time window and employ under timetable constraints. They study about flexible time window and heterogeneous repairman. Yuan et al. (2022) study on decentralized of the foundation of the model in heterogeneous environments. Yin et al. (2023) investigates on a pre-emptive two priority queueing system with the behavior of impatient customers and heterogeneous servers. This paper presents a queueing model with multi priority and two kinds of impatient customers. Aldea et al. (2023) study on real time monitoring and management hardware and software resources represent and ensure the security and reliability of the heterogeneous computer server. Saravanan et al. (2023) investigate on the concept of admission control policy of a two heterogeneous server with finite capacity retrial queueing system with maintenance activity. If server is available, then arrival customer immediately receives the service otherwise customers have to wait in orbit for service. Niranjan, and Latha, (2024) investigate on two-phase heterogeneous and batch service model in queueing system with breakdown in two-phase along with feedback and vacation. This study provides split services first essential service and the second essential service. A study of different server with different customer's behaviors is presented by Ayyappan and Thilagavathy, (2024). In this study a single server queue in which customer arrive in Markovian arrival process and correspondence with this service is phase-type distribution. Next review is on reboot policy in machine repair models.

Reboot is a powerful tool to manage real time of any kind of machine, through reboot machines can define their work easily and can meet up with goal in real time. Jain et al. (2020) investigate the maintainability issues in machining system during vacation, fault detection and delay in reboot. This study is about a multi-components system and spares are also used to increase the availability. Jain et al. (2021) study on the fuzzy Markovian modeling of machine system with imperfect coverage, spare provisioning and reboot policy. In this paper author examine the parametric non-linear programming method for the assessment of reliability, availability and maintainability. Kumar et al. (2022) considered the fuzzy model control based service on N optimal for fault tolerance system with reboot and recovery by using harmony search algorithm. The examination of failures of machines and reboot recovery process are taken into performance indices. Rani et al. (2023) investigate on optimization of fault-tolerant system with reboot, recovery and vacationing server under the admission of failed units for repair according to F-control policy and queueing model. When the failed machines set to the maximum capacity the system does not allow entering the failed machines until the drop down of failed machines to the bottleneck level Jain et al. (2024) analyzed the reliability and cost optimization of fault tolerant system with reboot and service interruption. Working vacation also play a major role in work efficiency of machining system. Raychaudhuri and Jain (2025) analyzed the effectiveness of retrial queueing system under threshold policy and vacation on servers.

Working vacation and the high reliability, efficiency without interruption are the key factor of real time machining system. Ketema et al. (2021) investigate in a M/M/2 machine repair model with controlled policy with multiple working vacations and triadic (0, Q, N, M). Cost analysis is done by Deora et al. (2021) in this study obtained the cost optimization of machine repair system with working vacation with feedback and particle swarm optimization. Thakur et al. (2021) and Jain et al. (2022) study on ANFIS and cost optimization for Markovian Queue with Operational Vacation, upgrading service rate with N-policy. Ahuja et al. (2022) and Bouchentouf et al. (2022) considered the multi-station unreliable machine system with working vacation policy and customer's impatient behaviour Kumar et al. (2023), Thakur and Jain (2024) study the double retrial system for repair facility features and fault tolerant machining system with working vacation. Multiple working vacation model are used in scenarios when the service station continues in operation at lower rate rather than completely down. Working vacation also plays an important role in any repair system to save energy and efficiency. Mehta et al. (2024) analyzed the Markovian unreliable retrial queue with different type of vacations along with ANFIS. Thakur and Jain (2025) considered Markovian queueing model under Bernoulli vacation and servers and obtained results to comparison by soft computing technique.

This paper presents a comprehensive investigation into the intricate dynamics of M/M/2 a heterogeneous system associated as the different type of services like essential services, optional services, priority services with working vacation. The rest of the paper is organized as follows. Section 2 provides the assumptions and notations for developing the concerned system. In next section 3 the explained some practical application of the proposed model. In the section 4, mathematical analysis and governing equations have been given. Section 5 provides performance measures of the system. Cost function is disguised in section 6. Results have been discussed in section 7. At last, a conclusion was given.

THE ASSUMPTIONS AND NOTATIONS

In this study considered Markovian queueing model with two heterogeneous servers and working vacation. This proposed model is described with suitable notation and assumptions.

Table 1: Notation and abbreviation used in model

Notations	**Abbreviation**
WV	Working vacation
λ_n	Rate of arrival of failed units
$\vartheta\lambda_n$	Rate of arrival of failed units for priority service
$\gamma\lambda_n$	Rate of arrival of failed units form WV to ES
μ	Service rate during ES state
μ_1	Service rate during optional service
μ_2	Service rate during PS service
μ_3	Service rate during working vacation
φ	Switching rate from PS to WV
φ_1	Rate of switching from ES state to OS service
φ_2	Rate of switching from OS to PS service
θ_1	Rate of switching from OS state to ES service
θ_2	Rate of switching from PS to OS service
θ	Rate of switching of ES state to WV
$\lambda_n\bar{b}$	Rate of switching in reboots state
ω	Recapture from reboot rate of the system.
α	Rate of breakdown of ES state
α_1	Rate of breakdown of OS state
α_2	Rate of breakdown of PS state (OS)
β	Repair rate of ES
β_1	Repair rate of OS state
β_2	Repair rate of PS state
$P_{n,0}$	The probability of ‘n’ failed units in working vacations state.
$P_{n,1}$	The probability that the server in reboot state of with ‘n’ failed units (FU)
$P_{n,2}$	The probability that the server in essential services state of with ‘n’ FU
$P_{n,3}$	The probability that the server in optional services state of with ‘n’ FU
$P_{n,4}$	The probability that the server in priority services state of with ‘n’ FU

$P_{n,5}$	The probability that the server in breakdown of priority services state of with 'n' FU
$P_{n,6}$	The probability that the server in breakdown of optional services state of with 'n' FU
$P_{n,7}$	The probability that the server in breakdown of essential services state of with 'n' FU
WV	Working vacation

A queueing system with two heterogeneous servers is considered. The first server is a cobot, this is the server where robots and humans jointly provide service. These cobot provide two services first service is essential service (ES) which is provided with service rate μ and the second service is optional service (OS) with service rate μ_1 are exponentially distributed. The second server is providing priority services and constantly working with service rate μ_2 based on exponential distribution. Reboot and recapture facility is also provided to avoid any situation if the system faces any problem in real-time management. Reboot rate of the system is $\lambda_n \bar{b}\ here\ \bar{b}\ as\ (1-b)$ and the recapture rate is ω .To develop the finite Markov model, the failed units (FS) are supposed to join the queue for service, arrival of failed units with rate λ follows the Poisson distribution. The first server in Essential Services (ES) provided combined services for human arrivals and failed units with a rate of $\lambda_n b$. In this approach it provided two services first is computer diagnostics and cure service which is necessary, by using this device. For further services, it can be pass to the next optional services (OS) with arrival rate λ_n based on Poisson distribution. The second services provide the parts upgrades of units (machines), custom modification of units (machines), performance monitoring, extended warranty, remote diagnostic, special cleaning etc.. If these services are required, then this second server provides the optional services (OS). The second server offers priority services, and in the case of priority services, the failed units arrive with rate $\vartheta\lambda_n$and the units can leave the server after receiving service at rate μ_2. The repair probability as p for both the essential services and the optional services with rate α, α_1 respectively and β, β_1 is the repair rate of essential and optional services respectively. Switching probabilities are $\bar{p}$ = (1-p) for essential to optional services and optional to priority services with rates φ_1, φ_2 respectively. a_2 is the rate of breakdown to priority services and its switching rate of repair is B_2. This is a P-policy, when the capacity (L) of the system is full, the failed machines are not allowed to enter inside, they have to wait outside until the count of failed statements falls below the threshold value "P". Every failed statement is served according to the first come first served (FCFS) rule. N(T) represents the count of failed statements present in the system at time t. S(t) is used to represent the state of the server during time t and is as follows-

PRACTIAL APPLICATION OF THE MODEL

This is a performance predication of the system which is applied in a hospital's queues and in the treatment of failed machines it plays a vital role in any system or at any platform, they are the main performance predication of any organization. To cite a practical application of the proposed model, giving an example of a general physician in a hospital. Patients are standing in queue for the waiting area, they are waiting for their turn or for being served. The integrated part of this system is the first server which provides two services the first service is essential service (ES) as robotic checkup, diagnostic and doctor's prescriptions, and the second service is optional service (OS) like health insurance, full one-year package of services etc.. In this model the first server doctor checkup the patients and prescribes the medicines according to their symptoms and patient left. But during the checkup of some patient doctor need some keen observation, so that doctor prescribes the patient some test to diagnose his disease. In that case the server plays its role by providing the essential robotic services, such as robots assisting in the diagnostic procedure, like taking blood for biopsy, or perform performing endoscopy, which helps in diagnosis. After the robotic checkup, if patient needs any specialist physician based on the diagnosis, this second server provides its optional services (OS) on the request of the patient. If patient don't want to get optional service it can revisit the doctor for further follow-up after diagnostic. The patient again visited the doctor and this time his disease is diagnosed then doctor prescribes him medicines and then patient left. The hospital has the sufficient capacity for patients is the threshold policy- L, more than L patient have to wait in waiting area. This server is associated with working vacation or emergency break to attend the ICU patient. During the WV it can be checking the IP patient or discharges the patients. As the patient arrive at the busy state and if the doctor (first server) is at WV so it can rejoin in busy state it depends on the health of patient. Second server provides the priority services for emergency patients. This study is the investigation of healthcare system, in any machine repair syndicate. the analytical equation for the transient state probability of two heterogeneous servers with working vacation.

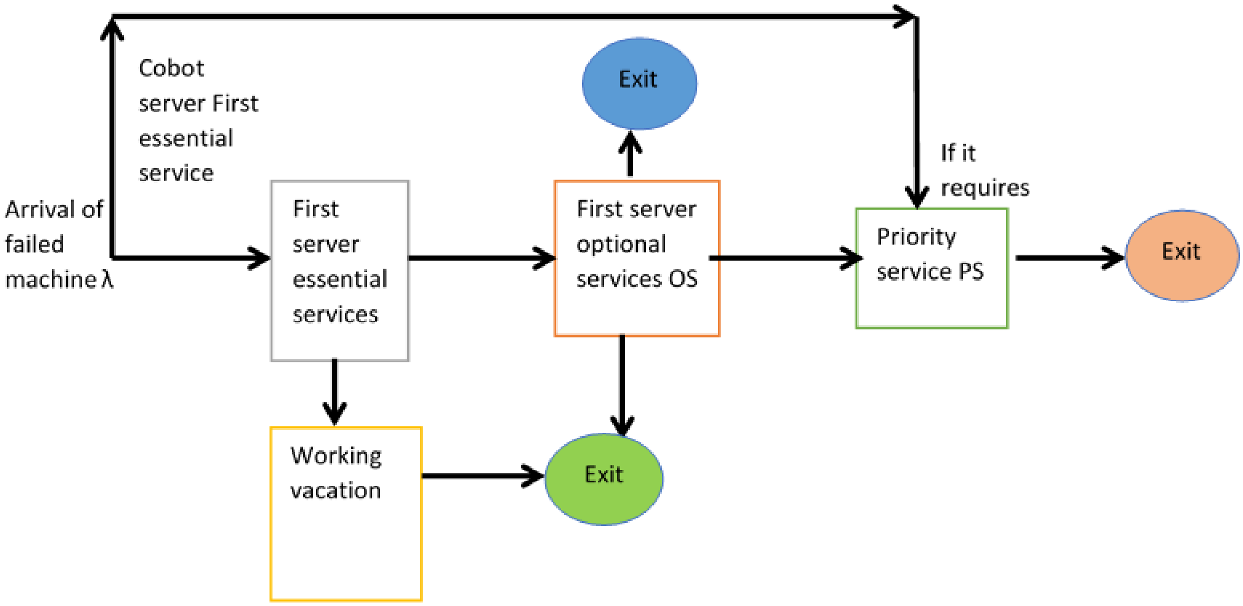

Fig. 1: Flow chart of the developed model

MATHAMATICS ANALYSIS OF THE MODEL

A mathematical analysis of the model for heterogeneous servers for two types of services essential service and optional service with repairable system is described in figure 1. The machine repair system is very complex with many uses that can be considered in variables, demand and resource. In the mathematical model, factors such as the arrival rate of failed units, service rate and other parameters are dynamically represented. Mathematical analysis is performed to understand the model and optimize the performance of the machine repair system. State of the system is defined as (ξ(n), x(i)), here the random variable n is representing the number of failed units in the system and the variable i represent the state of repair system.

$$i = \begin{cases} 0; & \textit{is the state of working vacation}(WV) \\ 1; & \textit{is the reboot state of the system} \\ 2; & \textit{is the essential service}(ES)\ \textit{state of the repair system} \\ 3; & \textit{is the optional service}(OS)\textit{state of repairsystem} \\ 4; & \textit{is the priority service}(PS)\ \textit{state of the repair system} \\ 5; & \textit{is the state of breakdown of priority service} \\ 6; & \textit{is the state of breakdown ofoptional services}(OS) \\ 7; & \textit{is the state of breakdown of essential server}(ES) \end{cases}$$

The steady -state of the probability is defined as;

$Л_{n,i}$ = probability of n($0 \leq n \leq K$) failed units in the repair system and i is the state of i($0 \leq i \leq 7$) server. The initial condition

$$Л_{n,i} = \begin{cases} 1; & if\ n = i = 0 \\ 0; & if\ 1 \leq n \leq K\ and\ 0 \leq i \leq 7 \end{cases}$$

The steady-state equations are developed by the inflow and outflow for every state depicted in the state transition figure 2. Differential-difference equations are-

i=0 for the working vacation state of the system;

$\frac{dЛ_{0,0}(t)}{dt} = -(\lambda_0 + \vartheta\lambda_0)Л_{0,0}(t) + \theta Л_{1,2}(t) + \mu_3 Л_{1,0}(t) + \varphi Л_{1,4}(t)$; n=1,

$$\frac{dЛ_{n,0}(t)}{dt} = -(\lambda_n + \mu_3)Л_{n,0}(t) + \lambda_{n-1}Л_{n-1,0}(t) + \mu_3 Л_{n+1,0}(t); 1 \le n \le N-1$$

$$\frac{dЛ_{n,0}(t)}{dt} = -(\lambda_N + \gamma\lambda_N + \mu_3)Л_{n,0}(t) + \lambda_{N-1}Л_{N-1,0}(t); N = n \le K$$

i=1 the reboot state of the repair system

$$\frac{dЛ_{1,1}(t)}{dt} = -(\omega)Л_{1,1}(t) + (\lambda_1\bar{b})Л_{1,2}(t)$$

$$\frac{dЛ_{n,1}(t)}{dt} = -(\omega)Л_{n,1}(t) + (\lambda_n\bar{b})Л_{n,2}(t);\ 2 \le n \le K-1$$

i=2 the essential service state of first server of the repair system

$$\frac{dЛ_{1,2}(t)}{dt} = -(\lambda_1 b + \lambda_1\bar{b} + \varphi_1\bar{p} + \alpha p + \theta)Л_{1,2}(t) + (\theta_1)Л_{1,3}(t) + (\beta)Л_{1,7}(t) + \mu Л_{2,2}(t)$$

$$\frac{dЛ_{n,2}(t)}{dt} = -(\lambda_n b + \lambda_n\bar{b} + \varphi_1\bar{p} + \mu + \alpha p)Л_{n,2}(t) + (\theta_1)Л_{n,3}(t) + (\beta)Л_{n,7}(t) + \mu Л_{n+1,2}(t) + \omega Л_{n-1,1}(t) + \lambda_{n-1}b\, Л_{n-1,2}(t); 2 \le n \le N-1$$

$$\frac{dЛ_{n,2}(t)}{dt} = -(\lambda_n b + \lambda_n\bar{b} + \varphi_1\bar{p} + \mu + \alpha p)Л_{n,2}(t) + (\theta_1)Л_{n,3}(t) + (\beta)Л_{n,7}(t) + \mu Л_{n+1,2}(t) + \gamma\lambda_n Л_{n,0}(t) + \omega Л_{n-1,1}(t) + \lambda_{n-1}b\, Л_{n-1,2}(t); n = N$$

$$\frac{dЛ_{n,2}(t)}{dt} = -(\lambda_n b + \lambda_n\bar{b} + \varphi_1\bar{p} + \mu + \alpha p)Л_{n,2}(t) + (\theta_1)Л_{n,3}(t) + (\beta)Л_{n,7}(t) + \mu Л_{n+1,2}(t) + \gamma\lambda_n Л_{n,0}(t) + \omega Л_{n-1,1}(t) + \lambda_{n-1}b\, Л_{n-1,2}(t); N+1 \le n \le K$$

i=3 the optional services state of first server of the repair system

$$\frac{dЛ_{1,3}(t)}{dt} = -(\lambda_1 + \varphi_2(\bar{p}) + \theta_1 + \alpha_1 p)Л_{1,3}(t) + (\theta_2)Л_{1,4}(t) + \varphi_1(\bar{p})Л_{1,2}(t) + (\beta_1)Л_{1,6}(t) + \mu_1 Л_{2,3}(t); n = 1$$

$$\frac{dЛ_{n,3}(t)}{dt} = -(\lambda_n + \varphi_2(\bar{p}) + \theta_1 + \alpha_1 p + \mu_1)Л_{n,3}(t) + (\theta_2)Л_{n,4}(t) + \varphi_1(\bar{p})Л_{n-1,2}(t) + (\beta_1)Л_{n,6}(t) + \mu_1 Л_{n+1,3}(t) + \lambda_{n-1}Л_{n-1,3}(t); 2 \le n \le K$$

i=4 the priority services state of second server of the repair system

$$\frac{dЛ_{1,4}(t)}{dt} = -(\vartheta\lambda_1 + \varphi + \theta_2 + \alpha_2)Л_{1,4}(t) + (\vartheta\lambda_0)Л_{0,0}(t) + \varphi_2(\bar{p})Л_{1,3}(t) + (\beta_2)Л_{1,5}(t) + \mu_2 Л_{2,4}(t); n = 1$$

$$\frac{dЛ_{n,4}(t)}{dt} = -(\vartheta\lambda_n + \theta_2 + \alpha_2 + \mu_2)Л_{n,4}(t) + (\vartheta\lambda_{n-1})Л_{n-1,4}(t) + \varphi_2(\bar{p})Л_{n,3}(t) + (\beta_2)Л_{n,5}(t) + \mu_2 Л_{n+1,4}(t); 2 \le n \le K$$

i=5 the priority services breakdown state of second server of the repair system

$$\frac{dЛ_{1,5}(t)}{dt} = -(\beta_2)Л_{1,5}(t) + \alpha_2 Л_{1,4}(t); n = 1$$

$$\frac{dЛ_{n,5}(t)}{dt} = -(\beta_2)Л_{n,5}(t) + \alpha_2 Л_{n,4}(t); 2 \le n \le K$$

i=6 the optional services breakdown state of first server of the repair system

$$\frac{dЛ_{1,6}(t)}{dt} = -(\beta_1)Л_{1,6}(t) + \alpha_1 pЛ_{1,3}(t); n = 1$$

$$\frac{dЛ_{n,6}(t)}{dt} = -(\beta_1)Л_{n,6}(t) + \alpha_1 pЛ_{n,3}(t); 2 \le n \le K$$

i=7 the essential services breakdown state of first server of the repair system

$$\frac{dЛ_{1,7}(t)}{dt} = -(\beta)Л_{1,7}(t) + \alpha pЛ_{1,2}(t); n = 1$$

$$\frac{dЛ_{n,7}(t)}{dt} = -(\beta)Л_{n,7}(t) + \alpha pЛ_{n,2}(t); 2 \le n \le K$$

Steady-state equations of $Л_{n,i}(t)$ in Laplace-Stieltjes transformation (LST) define as

$$Л^*_{n,i}(s) = \int_0^\infty e^{-st}\, Л_{n,i}(t)dt$$

The transformed equations are by taking LST given below;

i=0 the working vacation state of the repair system;

$$Л_{0,0}(0) = (\lambda_0 + \vartheta\lambda_0 + s)Л^*_{0,0}(s) - \theta Л^*_{1,2}(s) - \mu_3 Л^*_{1,0}(s) - \varphi Л^*_{1,4}(s)$$

$$Л_{n,0}(0) = (\lambda_n + \mu_3 + s)Л^*_{n,0}(s) - \lambda_{n-1}Л^*_{n-1,0}(s) - \mu_3 Л^*_{n+1,0}(s) \quad ; 1 \le n \le N-1$$

$$Л_{n,0}(0) = (\lambda_{n+1} + \gamma\lambda_n + \mu_3 + s)Л^*_{n,0}(s) - \lambda_{n-1}Л^*_{n-1,0}(s); N \le n \le K$$

i=1 the reboot state of the repair system

$$Л_{1,1}(0) = (\omega + s)Л^*_{1,1}(s) - (\lambda_1\bar{b})Л^*_{1,2}(s)$$

$$Л_{n,1}(0) = (\omega + s)Л^*_{n,1}(s) - (\lambda_1\bar{b})Л^*_{n,2}(s);\ 2 \le n \le K$$

i=2 the essential service state of first server of the repair system

$$Л_{1,2}(0) = (\lambda_1 b + \lambda_1\bar{b} + \varphi_1(\bar{p}) + \alpha p + \theta + s)Л^*_{1,2}(s) - (\theta_1)Л^*_{1,3}(s) - (\beta)Л^*_{1,7}(s) - \mu Л^*_{2,2}(s)$$

$$Л_{n,2}(0) = (\lambda_n b + \lambda_n\bar{b} + \varphi_1(\bar{p}) + \mu + \alpha p + s)Л^*_{n,2}(s) - (\theta_1)Л^*_{n,3}(s) - (\beta)Л^*_{n,7} - \mu Л^*_{n+1,2}(s) - \omega Л^*_{n-1,1}(s) - \lambda_{n-1}bЛ^*_{n-1,2}(s); 2 \le n \le N-1$$

$$Л_{n,2}(0) = (\lambda_n b + \lambda_n\bar{b} + \varphi_1(\bar{p}) + \mu + \alpha p + s)Л^*_{n,2}(s) - (\theta_1)Л^*_{n,3}(s) - (\beta)Л^*_{n,7}(s) - \mu Л^*_{n+1,2}(s) - \gamma\lambda_n Л^*_{n,0}(s) - \omega Л^*_{n-1,1}(s) - \lambda_{n-1}bЛ^*_{n-1,2}(s); n = N$$

$$Л_{n,2}(0) = (\lambda_n b + \lambda_n\bar{b} + \varphi_1(\bar{p}) + \mu + \alpha p + s)Л^*_{n,2}(s) - (\theta_1)Л^*_{n,3}(s) - (\beta)Л^*_{n,7} - \mu Л^*_{n+1,2}(s) - \omega Л^*_{n-1,1}(s) - \gamma\lambda_n Л^*_{n,0}(s) - \lambda_{n-1}bЛ^*_{n-1,2}(s); N+1 \le n \le K$$

i=3 the optional services state of first server of the repair system

$$Л_{1,3}(0) = (\lambda_1 + \varphi_2(\bar{p}) + \theta_1 + \alpha_1 p + s)Л^*_{1,3}(s) - (\theta_2)Л^*_{1,4}(s) - \varphi_1(\bar{p})Л^*_{1,2}(s) - (\beta_1)Л^*_{1,6} - \mu_1 Л^*_{2,3}(s); n = 1$$

$$Л_{n,3}(0) = (\lambda_n + \varphi_2(\bar{p}) + \theta_1 + \alpha_1 p + \mu_1 + s)Л^*_{n,3}(s) - (\theta_2)Л^*_{n,4}(s) - \varphi_1(\bar{p})Л^*_{n-1,2}(s) - (\beta_1)Л^*_{n,6}(s) - \mu_1 Л^*_{n+1,3}(s) - \lambda_{n-1}Л^*_{n-1,3}(s); 2 \le n \le K$$

i=4 the priority services state of second server of the repair system

$$Л_{1,4}(0) = (\vartheta\lambda_1 + \varphi + \theta_2 + \alpha_2 p + s)Л^*_{1,4}(s) - (\vartheta\lambda_0)Л^*_{0,0}(s) - \varphi_2(\bar{p})Л^*_{1,3}(s) - (\beta_2)Л^*_{1,5}(s) - \mu_2 Л^*_{2,4}(s); n = 1$$

$$Л_{n,4}(0) = (\vartheta\lambda_n + \theta_2 + \alpha_2 P_3 + \mu_2 + s)Л^*_{n,4}(s) - (\vartheta\lambda_{n-1})Л^*_{n-1,4}(s) - \varphi_2(\bar{p})Л^*_{n,3}(s) - (\beta_2)Л^*_{n,5}(s) - \mu_2 Л^*_{n+1,4}(s); 2 \le n \le K$$

i=5 the priority services breakdown state of second server of the repair system

$Л_{1,5}(0) = (\beta_2 + s)Л^*_{1,5}(s) - \alpha_2 p Л^*_{1,4}(s); n = 1$

$Л_{n,5}(0) = (\beta_2 + s)Л^*_{n,5}(s) - \alpha_2 p Л^*_{n,4}(s); 2 \le n \le K$

i=6 the optional services breakdown state of first server of the repair system

$Л_{1,6}(0) = (\beta_1 + s)Л^*_{1,6}(s) - \alpha_1 p Л^*_{1,3}(s); n = 1$

$Л_{n,6}(0) = (\beta_1 + s)Л^*_{n,6}(s) - \alpha_1 p Л^*_{n,3}(s); 2 \le n \le K$

i=7 the essential services breakdown state of first server of the repair system

$Л_{1,7}(0) = (\beta + s)Л^*_{1,7}(s) - \alpha p Л^*_{1,2}(s); n = 1$

$Л_{n,7}(0) = (\beta + s)Л^*_{n,7}(s) - \alpha p Л^*_{n,2}(s); 2 \le n \le K$

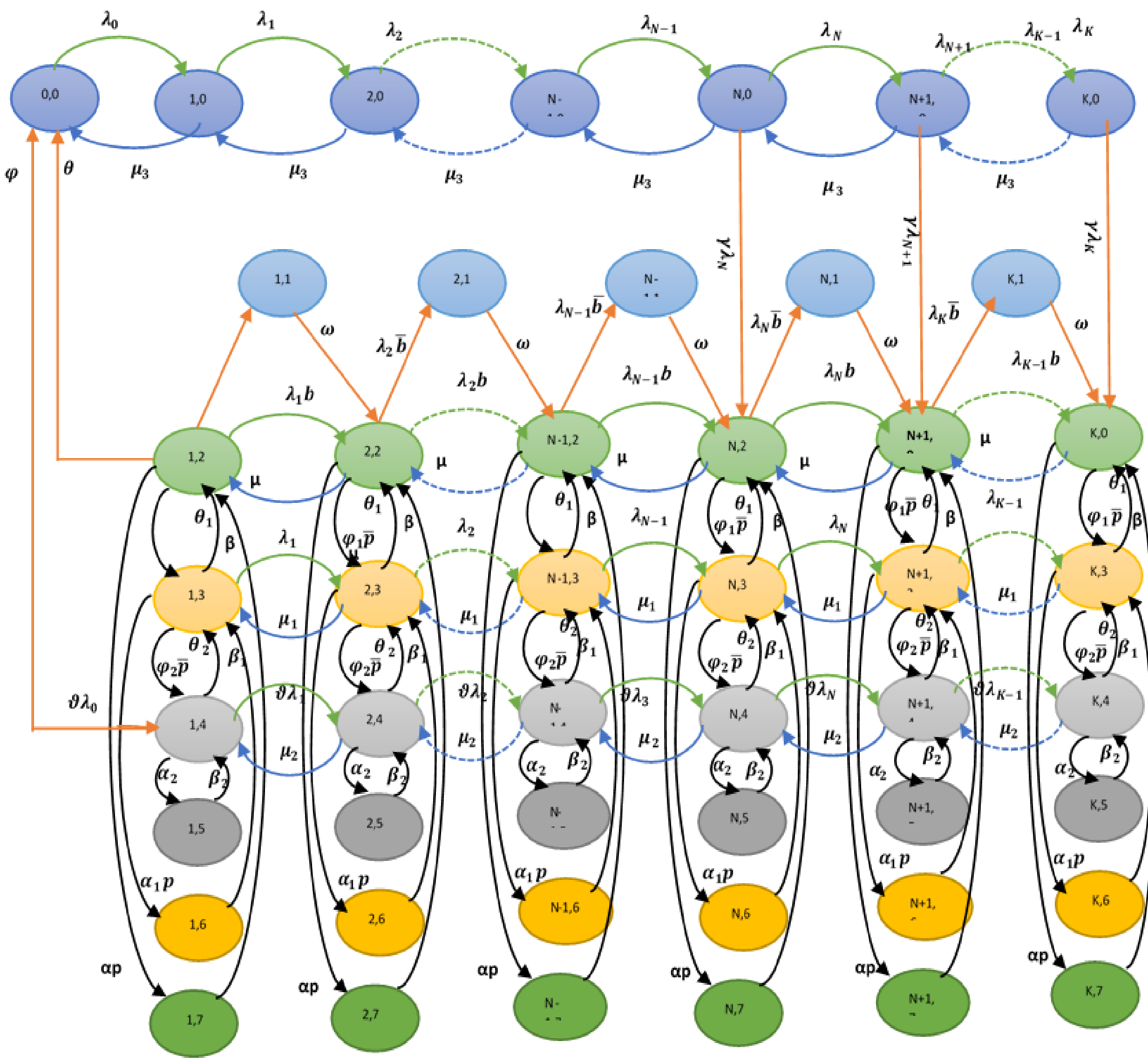

Fig.2 Transition rate diagram of the proposed model

PERFORMANCE MEASURES

The system state probabilities are adding to understand the distribution of failed units.

- SP (System Probability) when it is in working vacation state:

$$P_{WV} = \sum_{i=0}^{K} P_{i,0}(t)$$

- SP when it is in reboot state: $P_R = \sum_{i=0}^{K} P_{i,1}(t)$
- SP when it is in ES state: $P_{ES} = \sum_{i=0}^{K} P_{i,2}(t)$
- SP when it is in OS state: $P_{OS} = \sum_{i=0}^{K} P_{i,3}(t)$
- SP when it is in PS state: $P_{PS} = \sum_{i=0}^{K} P_{i,4}(t)$
- SP when it is in PS breakdown state: $P_{PSB} = \sum_{i=0}^{K} P_{i,5}(t)$
- SP when it is in OS breakdown state: $P_{OSB} = \sum_{i=0}^{K} P_{i,6}(t)$
- SP when it is in ES breakdown state: $P_{ESB} = \sum_{i=0}^{K} P_{i,7}(t)$

For evaluation of $P_{0,0}$ use the normalizing condition that is -

$$\sum_{i=0}^{K} P_{i,0} + \sum_{i=0}^{K} P_{i,1} + \sum_{i=0}^{K} P_{i,2} + \sum_{i=0}^{K} P_{i,3} + \sum_{i=0}^{K} P_{i,4} + \sum_{i=0}^{K} P_{i,5} + \sum_{i=0}^{K} P_{i,6} + \sum_{i=0}^{K} P_{i,7} = 1$$

Reliability Analysis of the System:

Let Y be the arbitrary variable that indicates the system's time to failure.

Where $P_{WV}(t) + P_R(t) + P_{ES}(t) + P_{OS}(t) + P_{PS}(t) + P_{PSB}(t) + P_{OSB}(t) + P_{ESB}(t)$ is the probability function is given by

$R_Y(t) = 1 - [P_{WV}(t) + P_R(t) + P_{ES}(t) + P_{OS}(t) + P_{PS}(t) + P_{PSB}(t) + P_{OSB}(t) + P_{ESB}(t)]$,

Average expected number of failed units in the system are-

- The EL (Expected Length) in the working vacation state

$$E(WV) = \sum_{n=0}^{K} n P_{n,0}$$

- The EL (Expected Length) in the reboot state

$$E(R) = \sum_{n=1}^{K-1} (n+1) P_{n,1}$$

- The EL (Expected Length) in essential services (ES) state

$$E(ES) = \sum_{n=1}^{K} n P_{n,2}$$

- The EL (Expected Length) in optional services (OS) state

$$E(OS) = \sum_{n=1}^{K} n P_{n,3}$$

- The EL (Expected Length) in priority services (PS) state

$$E(PS) = \sum_{n=1}^{K} n P_{n,4}$$

- The EL (Expected Length) in breakdown of priority services (PSB) state

$$E(PSB) = \sum_{n=1}^{K} n P_{n,5}$$

- The EL (Expected Length) in breakdown of optional services (OSB) state

$$E(OSB) = \sum_{n=1}^{K} n P_{n,6}$$

- The EL (Expected Length) in breakdown of essential services (ESB) state

$$E(ESB) = \sum_{n=1}^{K} n P_{n,7}$$

- The EL (Expected Length) waiting in the services area is $L_{(S)}$

$$L_S = \sum_{i=0(i\neq 1)}^{7} \sum_{n=0}^{K} nP_{n,i} + \sum_{n=1}^{K-1} (n+1)P_{n,2}$$

- The EL (Expected Length) waiting in the queue L_q is

$$L_q = \sum_{i=0(i\neq 1)}^{7} \sum_{n=0}^{K} (n-1)P_{n,i} + \sum_{n=1}^{K-1} (n)P_{n,2}$$

- The expected waiting time W_S of failed units in the system is

$$W_S = \frac{L_S}{\lambda_{eff}}$$

Where the $\lambda_{eff} = \sum_{n=0}^{K} \lambda_n (P_{n,0} + P_{n,2} + P_{n,3} + P_{n,4} + P_{n,5} + P_{n,6} + P_{n,7})$

Adaptive Neuro-Fuzzy Inference System (ANFIS) Based Model

ANFIS is a good controller. It is applicable in-home appliance, traffic management, telecommunication, weather information and in AI also. This is a compound of two techniques: first is artificial neural networks (ANN) and second is fuzzy system (FS). In the fuzzy play the if-then rule by this we can put input-output data and an ANFIS input output function also. The statement of IF U AND V THEN, where U and V are fuzzy set which needed of membership functions. There are three major requirements of fuzzy system- First to select the fuzzy rules, second to define the membership function form the data structure which is used in fuzzy rules. Third is logical mechanism, which performs the inference procedure based on the considered fuzzy rules in figure 3.

Fuzzy rules can be defining as-

- If u is U_1 and v is V_1 then$f_1 = p_1u + q_1v + r_1$.
- If u is U_2 and v is V_2 then$f_2 = p_2u + q_2v + r_2$.
- If u is U_3 and v is V_3 then$f_3 = p_3u + q_3v + r_3$.

Here U and V are the membership function for the x, y, z inputs and $p_1, p_2, p_3, q_1, q_2, q_3$ are the multifunction parameters.

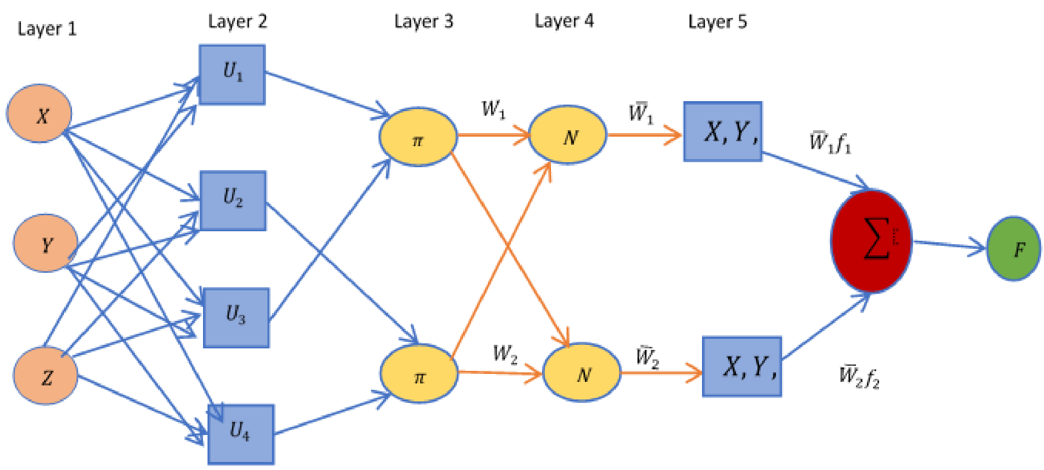

Fig.3 ANFIS Architecture

COST OPTIMIZATION

Once the model is developed and after completing its computational and its numerical simulation, next is to move towards the calculation of its cost factor. The cost function is used to calculate the optimal repair rate and total optimum cost of the system. The notation for the total cost function are defined as:

C_{wv}: Cost per unit time for working vacation present in the system.

C_R: Cost per unit time for failed units' present reboot state in the system.

C_{ES} : Cost per unit time for failed units in ES (essential service) state present in the system.

C_{OS}: Cost per unit time for failed units present in OS (optional service) state of system.

C_{PSB}: Cost per unit time for failed units present in PSB (breakdown of priority services) state

of the system.

C_{OSB}: Cost per unit time for failed units present in OSB (breakdown of optional services)

state of the system.

C_{ESB}: Cost per unit time for failed units present in ESB (breakdown of essential services)

state of the system.

C_μ:Cost per unit time while μ is the service rate.

$C_{\mu 1}$:Cost per unit time while μ_1 is the service rate.

$C_{\mu 2}$:Cost per unit time while μ_2 is the service rate.

$C_{\mu 3}$:Cost per unit in working vacation time while μ_3 is the service rate.

The expected total cost (TC) function of Model based on above parameters is

TC(F, μ,β)=$C_H L_S + C_{wv}P_{wv} + C_R P_R + C_{ES}P_{ES} + C_{OS}P_{OS} + C_{PS}P_{PS} + C_{ESB}P_{ESB} + C_{OSB}P_{OSB} + C_{PSB}P_{PSB} + C_\mu \mu + C_{\mu1}\mu_1 + C_{\mu2}\mu_2 + C_{\mu3}\mu_3$

Min TC(F, μ,β); $\mu,\beta \geq 0$ minimize the total cost by using the MATLAB software and ANFIS verification for optimization technique with respect to the μ is the service rate.

SENSITIVITY ANALYSIS

The numerical experiment is performed to investigate this heterogeneous M/M/2 model with the different parameters. As for calculating the performance measures fixing some parameters and a set of cost parameters. $M = 12, W = 3, K = 10, \lambda_n = (i+1)\lambda, \mu = 10, \mu_1 = 5,\ \mu_2 = 4, \mu_3 = 3, \varphi = 1.1, \varphi_1 = .5, \varphi_2 = .5, \theta = .5, \theta_1 = .3, \theta_2 = 10, \alpha = .5, \alpha_1 = .4\ , \alpha_2 = .3, \beta = 5, \beta_1 = 4,\ \beta_2 = 3,\ \omega = .3, p = .3, \vartheta = .5, \mathrm{C_H} = 50, \mathrm{C_{wv}} = 20,\ \mathrm{C_R} = 30,\ \mathrm{C_{ES}} = 15,\ \mathrm{C_{OS}} = 60,\ \mathrm{C_{PS}} = 40,\ \mathrm{C_{ESB}} = 10,\ \mathrm{C_{OSB}} = 45,\ \mathrm{C_{PSB}} = 35,\ \mathrm{C_\mu} = 50,\ \mathrm{C_{\mu1}} = 40,\ \mathrm{C_{\mu2}} = 30,\ \mathrm{C_{\mu3}} = 50$

Table 1 shows the membership function of linguistic values corresponding to the various input parameters. As essential service is defined as μ, the optional service rate defined as μ_1, the service provided as the priority service with rate μ_2 . μ_3 is the service rate during the working vacation state. Figure 3 presented the membership function of different variables. In this investigation the expected length is calculated with respect to the different rates in figure 5(a) to figure 5(f). The expected length (Ls) of the queue with parameter t is shown in figure 5(a) to be clearly exponential as it increases with the arrival rate (λ). Ls is calculated with respect to the probability (p) which shows a moderate change with time (t) in figure 5(b). Figure 5(c) shows L_s defined with respect to time (t) and the priority rate $\vartheta = .3\ to\ \vartheta = .9$ which changes gradually as t increasing. Figure 5(d) depict the L_s with respect to the $\omega = .5$ to $\omega = 1.1$ which is the reboot recapture rate which shows a slight variation with t. Figure 5(e) deliberate L_s with respect to the probability of customers leaving the system, it varies mercilessly with time. In fig. 5(f) L_s is represented with respect to the $\theta_2 = .5\ to\ \theta_2 = 2$ the switching rate from the priority to the essential service is increasing very slightly with time.

The reliability (RT) impact of different parameters is shown in Figures 6(a)-6(f). Figures 6(a) and 6(b) show the impact of failure rate (λ) and priority rate (ϑ) with respect to time (t) respectively which shows a decreasing trend.

In figure 6(c) and 6(d) display the effect of $\theta = .3\ to\ \theta = .9$ and repair probability p = 0.1 to p = 0.7, Rt is decreasing with both the parameters increases respectively by varying time (t). Fig 6(e) and 6(f) shows the outcomes of Rt with switching rate from priority to working vacation (φ) and reboot recapture rate (ω) slightly changes with time respectively. In figs. 7(a)-7(f) depict the 3D effect of cost set with different parameters by varying t. In fig. 7(a), 7(d) and 7(e) display the decreasing trend of μ, φ and μ_1 with cost set by increasing t. Fig. 7(b), 7(c) and 7(f) shows the increasing trend with parameters p, θ, and ω by varying t for some fixed cost set values.

In overall conclusion to display the effect of different parameters

(i) The queue length increasing with increasing the failure rate (λ), rate of switching in reboots state (b), reboot recapture rate (ω), rate of arrival of failed units for priority service (ϑ), and decreasing with rate of switching of essential service state to working vacation state (θ), rate of breakdown probability (p) with varying time (t) as in natural form. The similar effect shows with ANFIS.

(ii) The reliability gradually decreasing with failure rate (λ), and reboot recapture rate (ω), rate of arrival of failed units for priority service (ϑ) but the gradually increasing trend with switching of essential service state to working vacation state (θ), rate of breakdown probability (p) and switching rate from priority to working vacation (φ) by varying time (t) as usual effects.

(iii) The effect of total cost shown in figures and trend are displayed as real time.

CONCLUSIONS

In this study, a model of M/M/2 heterogeneous servers is developed with some different types of services such as Essential Services (ES), Optional Services (OS) and Priority Services (PS) with breakdown and repairs rates. The cost function is also calculated to make the proposed model more economical. Some soft-computing techniques based on optimization can also be added to this study to make it more futuristic and more sustainable. For further work it can be extended with retrial or double retrial, balking, reneging queueing system.

Table 1 Membership function and the corresponding linguistic values of the input parameters $\lambda_{n,}$ $\mu, p, \theta, \vartheta, \omega$

Input Variables	No. of Membership Functions	Linguistic Values
Arrival rate (λ)	6	Very Low; Low; Moderate; High; Very High
Service Rate in the busy state (μ)	6	Very Low; Low; Moderate; High; Very High
Recapture from reboot rate of the system (ω)	6	Very Low; Low; Moderate; High; Very High
Rate of switching of ES state to WV (θ)	6	Very Low; Low; Moderate; High; Very High
Probability for priority service (p)	6	Very Low; Low; Moderate; High; Very High
Switching rate from PS to WV (ϑ)	6	Very Low; Low; Moderate; High; Very High

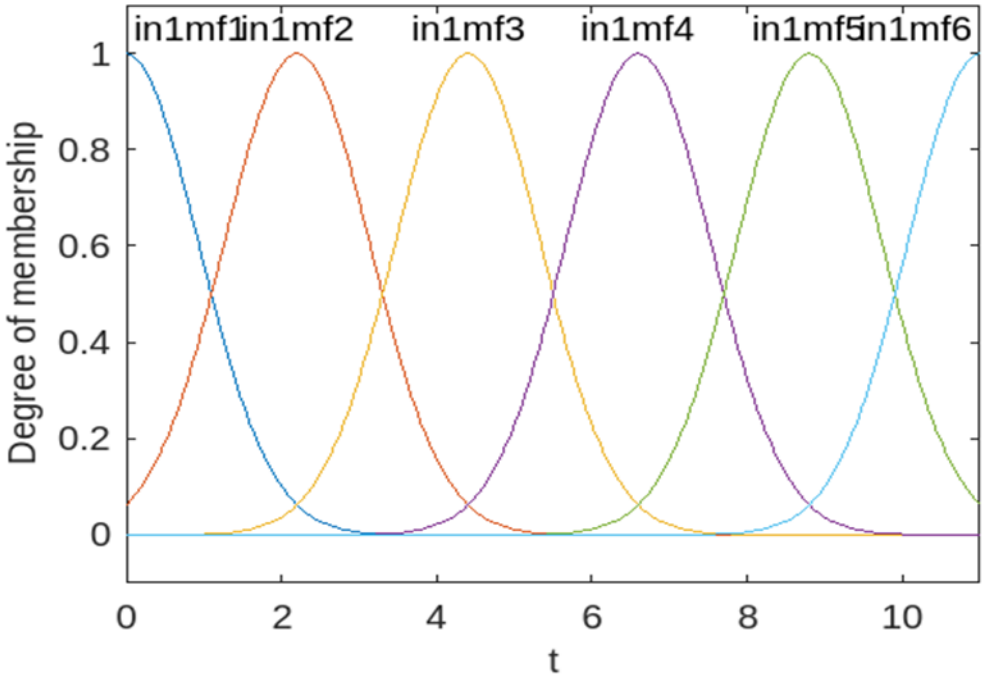

Fig. 4: Membership function for input variables

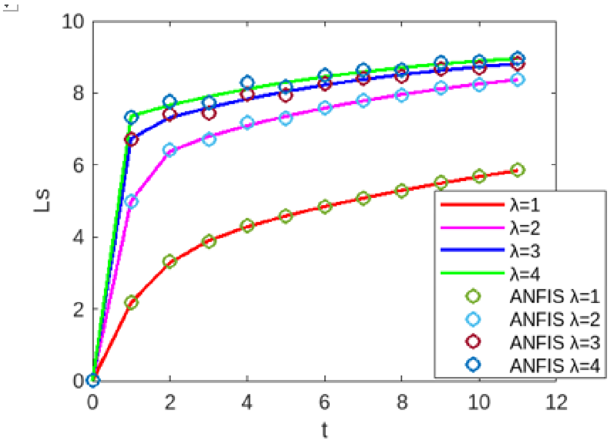

Fig. 5(a) Ls vs. time (t) varying by λ

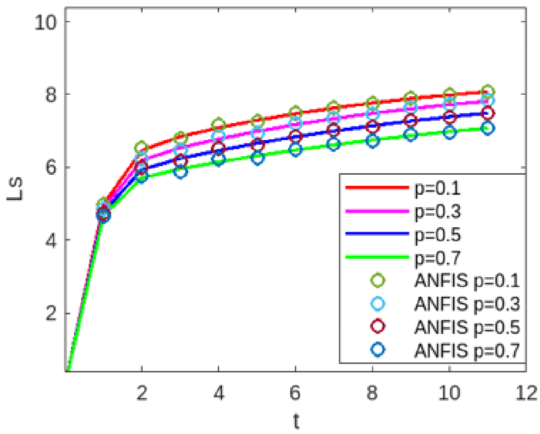

Fig. 5(b) Ls vs. time (t) varying by p

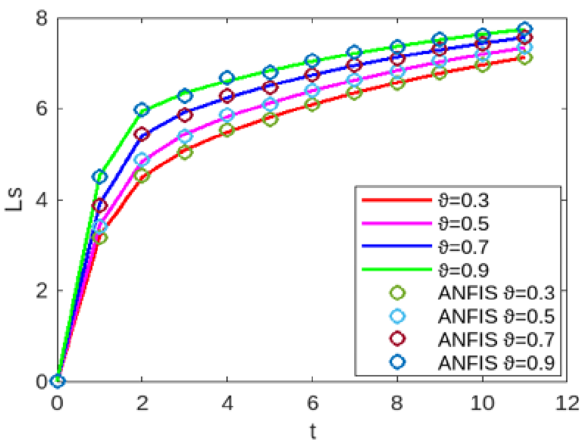

Fig. 5(c) Ls vs. time (t) varying by ϑ

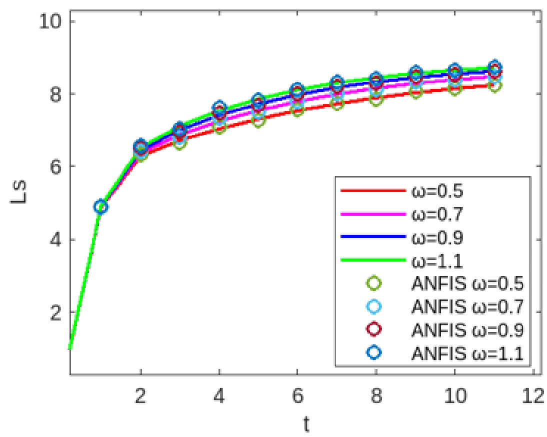

Fig. 5(d) Ls vs. time (t) varying by ω

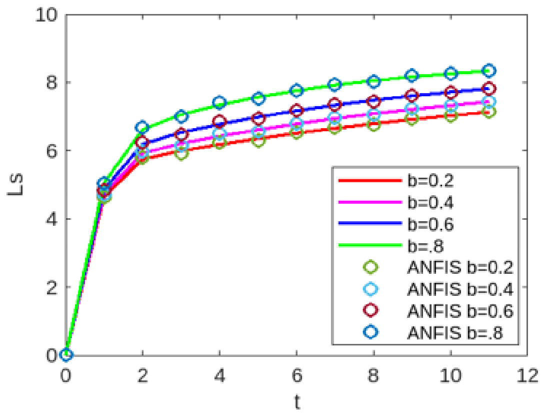

Fig. 5(e) Ls vs. time (t) varying by b

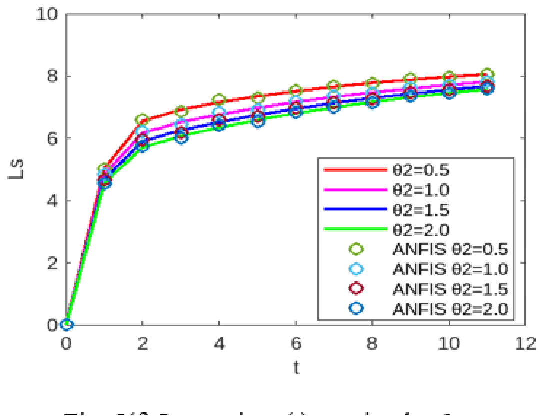

Fig. 5(f) Ls vs. time (t) varying by θ_2

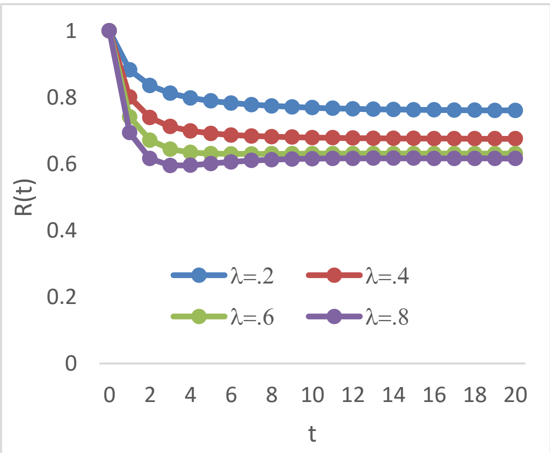

Fig. 6(a) Rt and the time varying by λ

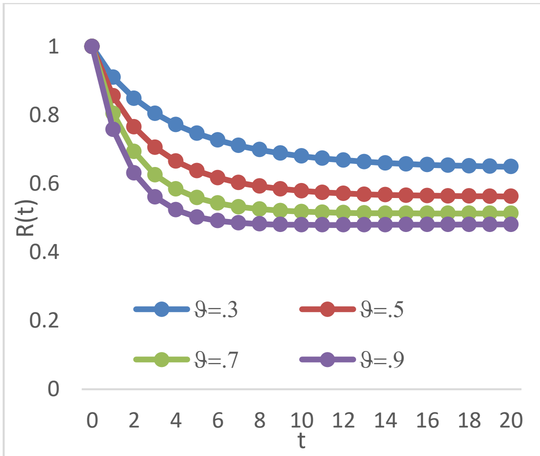

Fig. 6(b) Rt and the time varying by ϑ

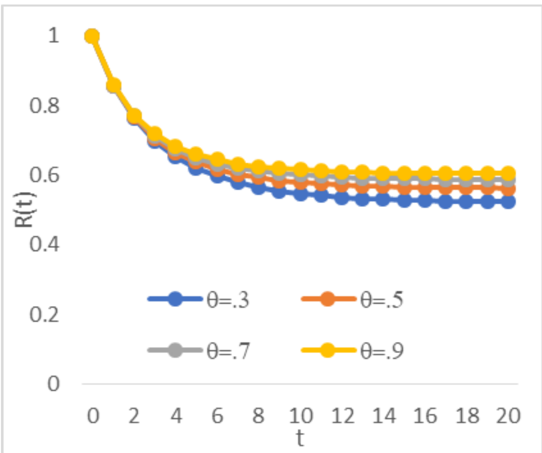

Fig. 6(c) Rt and the time varying by θ

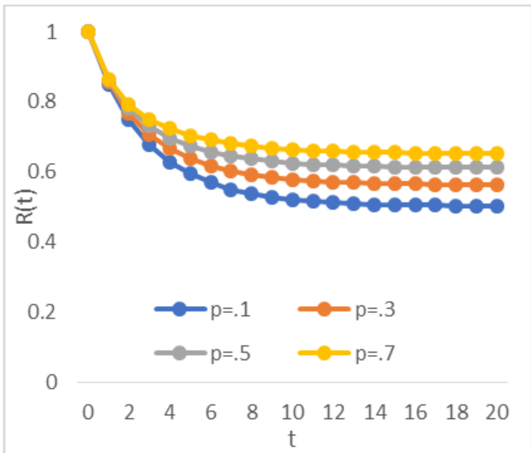

Fig. 6(d) Rt and the time varying by p

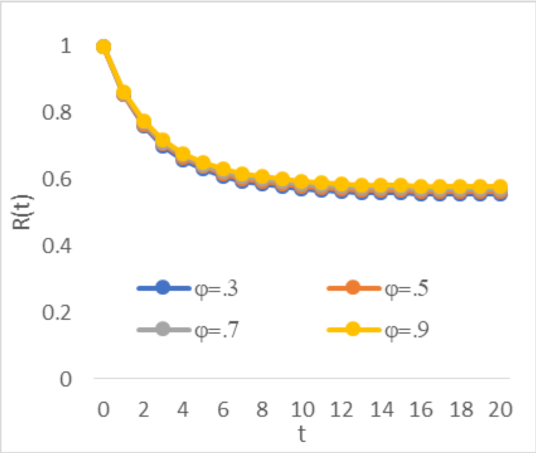

Fig. 6(e) Rt and the time varying by φ

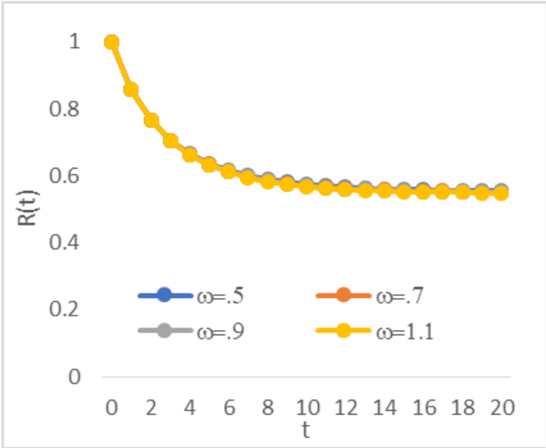

Fig. 6(f) Rt and the time varying by ω

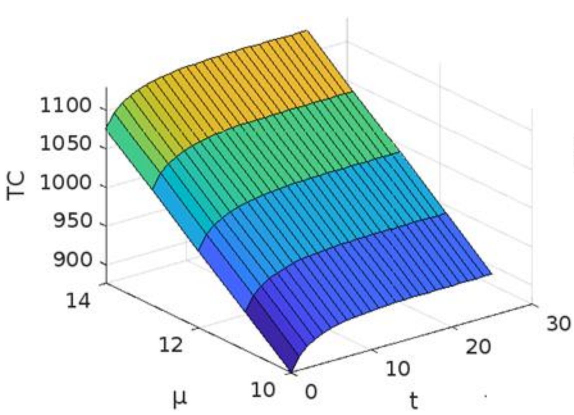

Fig. 7(a) TC and the time varying by μ

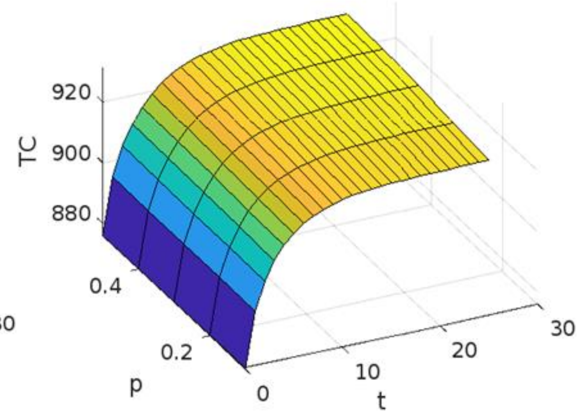

Fig. 7(b) TC and the time varying by p

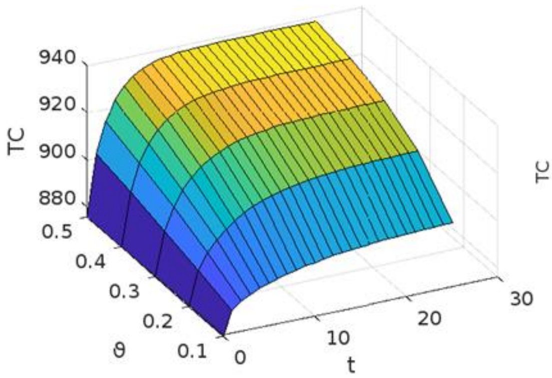

Fig. 7(c) TC and the time varying by θ

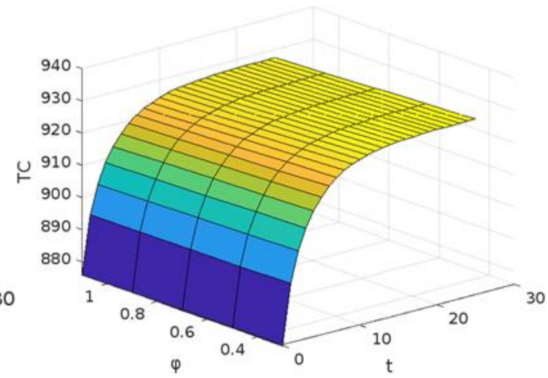

Fig. 7(d) TC and the time varying by φ

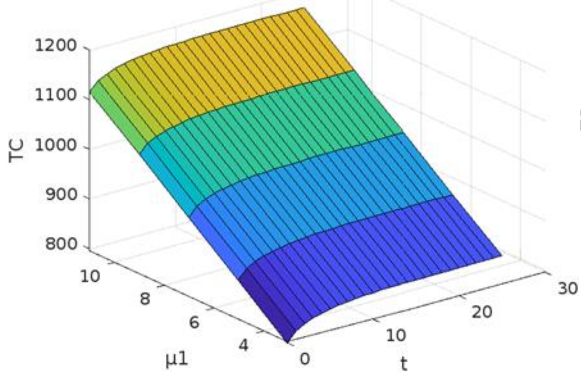

Fig. 7(e) TC and the time varying by $\mu 1$

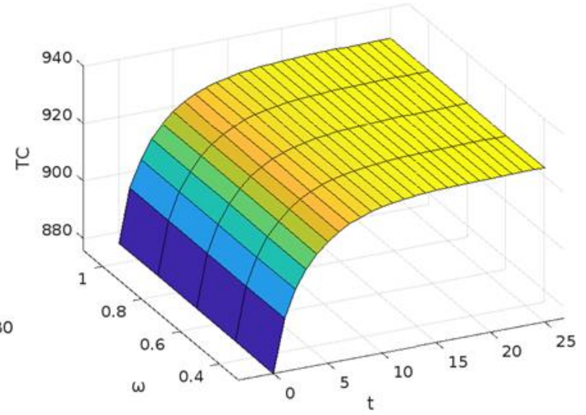

Fig. 7(f) TC and the time varying by ω

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